Optimal Management of Supply Disruptions when Contracting with Unreliable, Risk-averse, Suppliers

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WHEN CONTRACTING WITH UNRELIABLE, RISK-AVERSE, SUPPLIERS. [1]

Abstract: This paper investigates the optimal management of supply disruptions by a manufacturer who uses order inflation and/or investments in process reliability when contracting two risk-averse suppliers. We consider that these investments can be subject to moral hazard. Technically we solve a newsvendor optimization problem using a random capacity model of disruption. In such a model, the order size does not affect the average production but impacts the probability of disruption.

When investments are verifiable we show that the manufacturer is more inclined to invest in the suppliers' reliability and then refrain from using order inflation when the suppliers' production costs and the cost of disposing of unwanted inputs are large. When investments are not verifiable we show that the order sizes can be used strategically as incentive devises due to the suppliers' sensitivity to payoff dispersion. We show that the manufacturer does not always increase his reliance on order inflation and face less reliable suppliers once we introduce moral hazard. In some instances he induces suppliers to undertake larger investments in reliability by increasing the order size. In other instances he is able to reduce his reliance on order inflation.

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1. Introduction

Designing a profitable outsourcing strategy is a non-trivial task, as acknowledged by the substantial literature on the subject. It is complex because manufacturers are at least as concerned about managing ex-post contingencies and incentives as they are about addressing ex-ante information asymmetries (Bajari and Tadelis 2001). A survey conducted by Accenture (Erhardt et al. 2010) finds that "*innovation that improves reliability and quality*" and the "*reliability of supply*" are ranked above cost considerations when contracting with new or existing suppliers. Cohen et al. 2008 provides further evidence that such criteria can prevail over costs.

The most common proactive strategies used in practice to address reliability issues consist of inflating orders, investing in increased process reliability and diversifying the supply base. Yet, the sourcing strategies available to a manufacturer may be limited. For inputs produced by a large number of suppliers, the manufacturer can diversify his supply base and base the orders on each supplier's characteristics (Babich et al 2007, Dada et al 2007, Ferdergruen and Yang 2008 and 2009, Tang and Kouvelis 2011, Xu et al 2011 and more recently Yang et al 2015). But some inputs can only be produced by a few suppliers because of their complexity or because of quality standards. For these inputs, the manufacturer has to contract with designated suppliers and may lose the ability to diversify his supply base. To address reliability issues he can devote resources to improving the suppliers' performance. This strategy has been extensively documented in relation to the car industry under single or dual sourcing (Handfield et al. 2000, Wouters et al. 2007, Liker and Choi 2004). Krause 1997 and 1999, Krause et al. 1998 and 2007 provide empirical support for the supplier development strategies available to manufacturers.

The efficacy of each sourcing strategy depends on its impact on suppliers. To align objectives and incentivise suppliers, the manufacturer may be tempted to rely on penalties and rewards (Gurnani and Gerchak 2007, Yan et al. 2010 and Inderfurth and Clemens 2014). Whether such a strategy induces suppliers to improve their reliability depends on whether they are risk averse and thus reluctant to accept a greater dispersion of their profits. Using the Taiwanese automotive industry for data, Liu and Chen 2013 provide evidence that manufacturers absorb more risk when uncertainty

increases and when suppliers exhibit greater risk aversion. Focusing on the Japanese automotive industry, Tabeta and Rahman 1999 find that risk-averse suppliers are more likely to join a *keiretsu* organization in unfavourable business environments to avail insurance against fluctuations of their profits rate. Finally, and as established in the contract theory literature (see, for instance, Baron and Besanko 1987) profit dispersion is an important incentive device in the presence of risk-averse players.

In this paper we assume that the manufacturer can procure inputs from at most two risk-averse suppliers. We therefore leave diversification issues (beyond single vs. dual sourcing) aside. Instead, we emphasize the strategic use of order sizes, including order inflation, and incentive contracts to address supply disruptions when investments are and are not contractible. The research devoted to international trade shows that contractual incompleteness impacts the profitability of offshoring (Acemoglu et al. 2007 and Nunn 2007). The decision to purchase inputs from suppliers located in a country where wages are low can increase a manufacturer's competitive advantage provided the gains from paying lower wages are not eroded by costs associated with a lesser ability to enforce contracts. Understanding how moral hazard issues impacts the contracting cost is therefore particularly relevant in such settings.

Specifically, we analyse a newsvendor optimisation problem under supply uncertainty whereby each supplier's production is the realization of a random variable which follows an exponential distribution. Our modelling approach bears close resemblance to the capacity uncertainty models in that supply and order sizes are independent. The benefit of considering an exponential distribution is that the rate of occurrence parameter can be interpreted as a supplier's capacity, and thus reflects his reliability. While production is independent of the order size, the risk of a shortfall is positively correlated with the order size. Thus, we propose a model where risk and order sizes are positively correlated, reflecting situations where larger orders are subject to more risk either because they take longer to complete or because they require more intensive use of machinery. We then consider that the rate of the exponential distribution is endogenous and determined by some investment which may be subject to moral hazard. In the first part we assume that all variables are contractible and fully characterize the orders and investment decisions. This assumption enables us to bring to light the circumstances (i.e. price and cost structures) under which the manufacturer increases

his reliance on order inflation as opposed to increasing investments. In the second part we assume that investment is not contractible. We then analyse the impact that moral hazard has on optimal contracts. In particular we investigate whether the manufacturer relies more heavily on order inflation and deals with less reliable suppliers. The paper adopts a contract theory method whereby investments are undertaken by the suppliers but the manufacturer decides on a monetary transfer which covers the cost of such investments and, under moral hazard, incentivises the suppliers.

When investments are contractible, profit dispersion is sub-optimal. The suppliers are fully insured so that they are neither rewarded for completing the order nor penalized for failing to do so. Unsurprisingly, when reliability is heterogeneous, we show that the more reliable supplier receives larger orders. For any given rates of occurrence, order inflation is optimal for a wider range of parameters when the price of the manufacturer's output increases or when the cost associated with a shortage of input increases. In contrast, it is optimal for a narrower range of parameters when the suppliers' marginal cost increases or when the cost associated with an excess of inputs increases. We then show that the manufacturer is more inclined to invest in the suppliers' reliability, and then refrain from using order inflation, when the production costs and the cost of disposing of unwanted inputs are large. It is important to note that, in our setting, the manufacturer inflates order not because it impacts the average production (as it would in a random yield model) but because it reduces the risk that the overall production falls below the amount needed to satisfy the demand.

Once we introduce moral hazard, we show that the order size can be used strategically. Larger orders are subject to a greater risk of being incomplete. This would be inconsequential if the suppliers were, once again, fully insured. But to address moral hazard the manufacturer must rely on bonuses and penalties to incentivise the suppliers to undertake the appropriate investment. We then show that the optimal contract relies on fine-tuning the order sizes with some degree of profit dispersion. Orders that are below a supplier's capacity are more likely to be completed. In such a case, the manufacturer can induce higher investments by marginally increasing the order size without widening the profit dispersion, which is ultimately inefficient. The opposite holds for orders that are above a supplier's capacity and thus less likely to be completed.

Clearly, the introduction of moral hazard is costly and will not allow the manufacturer to reduce his reliance on order inflation and reach higher investments. However it is not clear that he will always have to increase his reliance on order inflation and face less reliable suppliers. While we do not characterize the optimal contract we show that, in some instances, the introduction of moral hazard leads the manufacturer to reduce his reliance on order inflation (but face less reliable suppliers). In other instances it leads the manufacturer to implement higher investments so as to face more reliable suppliers (but with a greater reliance on order inflation).

The structure of the paper is as follows. The next section reviews the literature. Section 3 presents the model. Section 4 characterises the optimal outsourcing strategy when investments are contractible. Moral hazard issues are introduced and analysed in section 5. Finally, we conclude in section 6.

2. Literature review

A vast literature is dedicated to the causes of and remedies to various forms of supply chain disruptions. Snyder et al. 2016 provide a useful, comprehensive survey of this literature. The large number of publications accounts for the facts that reliability can be modeled in different ways and that there are diverse approaches to address disruptions.

Reliability has been mostly incorporated assuming either random capacity (e.g. Ciarallo et al. 1994), or random yield (e.g. Gerchak and Parlar 1990) or random disruptions (e.g. Parlar and Perry 1996). In the random yield model, capacity is not an issue and production is a random fraction of the order. In the random capacity model, production is bounded above by capacity so that orders are incomplete when the suppliers' realized capacity falls below the order size. Finally, under random disruptions the suppliers are either able or unable to complete their orders (due, for instance, to the occurrence of a natural disaster).

The possible approaches manufacturers use to thwart supply disruptions can be split into proactive and reactive strategies. In this paper we are interested in the former as we consider investments that improve the supplier's reliability. Tomlin 2006 shows that

an optimal approach can actually consist in using a combination of proactive and reactive strategies.

Among the proactive measures, and as argued in the introduction, the manufacturer can rely on order inflation, invest in improved production processes and/or diversify his supply base. Optimal diversification has received much attention. See, for instance, Babich et al 2007, Dada et al 2007, Ferdergruen and Yang 2008 and 2009, Tang and Kouvelis 2011 and Xu et al 2011. Among these, Dada et al 2007 provide a very general model that encompasses all possible reliability models. In a recent paper, Yang et al 2015 contributed to this research strand considering a situation where reliability is private information. These authors then weight the benefits of diversification against competition. Our paper leaves diversification issues aside to shed more light on order inflation and investments.

The reliance on order inflation appears to make more sense when production follows a random yield given the assumed link between the suppliers' output and the order. This intuition finds support in Tang et al 2014 which shows that order inflation is never used when production is all-or-nothing. By opposition they show that it is used, in conjunction with subsidies, under partial disruptions. In this paper we show that order inflation can be optimal in a model where production and orders are not related. In our setting, the manufacturer may inflate the order of the more reliable supplier so as to reduce the risk of a shortfall in the overall production. Under moral hazard, and under certain circumstances, we show that increasing the order size can be used as an incentive devise.

Clearly, inflating orders means that more inputs must be produced and thus paid for. We show that the manufacturer is more inclined to invest in increased reliability (so as to decrease his reliance on order inflation) as the suppliers' production cost increase. Wang et al. 2010 reach a similar conclusion. They compare dual sourcing to process improvement in a random capacity and random yield model. However, what their analysis most emphasizes is the important impact that cost and reliability heterogeneity across suppliers have on the optimal strategy.

While we adopt a contract theory model we do not specify the form of transfers from the manufacturer to the supplier. In our approach these are constrained by a

participation and, under moral hazard, an incentive constraint. Yin and Ma 2015 compare unit bonus to a lump sum transfer as incentive tools in a random yield model. They find that both can increase welfare.

An important element of the analysis performed here is the consideration of risk aversion on behalf of the suppliers. As argued in the introduction, under such an assumption, the suppliers respond to increased transfers but also to profit dispersion resulting from the use of penalties and bonuses. As a result, we find that under moral hazard reliability may actually increase. This result resonates with findings in Starbird 1994 and Xie et al. 2011. Starbird 1994 considers a practice known as *acceptance sampling* used to monitor the suppliers' output quality. He shows that risk-averse suppliers, sensitive to profit dispersion, provide higher quality products. Xie et al. 2011 consider a make-to-order supply chain and shows that a risk averse supply chain (which includes the manufacturer) provides higher quality outputs when the supplier is a leader in a Stackelberg game and selects the quality prior to the manufacturer setting the price.

In conclusion we feel that this paper fills a niche by highlighting the role of order sizes and investments in reliability in a model where the orders have no direct impact on production but instead affect the risk of failure. In a context where suppliers are risk averse, we analyse the impact of moral hazard which requires the fine-tuning of orders and profit dispersion.

3. The Model

A risk neutral manufacturer has an order of size Q^* at unit price $p > 0$. For instance consider a car manufacturer who has set the price of each of his models and faces a demand of Q^* units for one of these models. Assume, without loss of generalities, that $Q^* = 1$. To address the demand, the manufacturer relies on the supply of a specific input. To simplify matters we assume that the technology used by the manufacturer is such that one unit of output requires one unit of input. Hence, the manufacturer must order a quantity at least equal to one to satisfy the demand. Let $Q^D = q_1^D + q_2^D$ denotes the overall quantity of input that is delivered and q_i^D is the quantity delivered by

supplier i. Using a newsvendor optimisation problem, we account for the following possibilities:

- When $Q^D < 1$ then the manufacturer can only produce Q^D units of output and he incurs an opportunity loss c_S associated with his inability to meet the demand.
- When $Q^D > 1$ the demand is satisfied but the manufacturer must discard the unused inputs at cost $c_D \geq 0$.

Specifically, the manufacturer's revenue is given by the following expression

$$R(Q^D) = p \min\{1, Q^D\} - c_S \max\{1 - Q^D, 0\} - c_D \max\{Q^D - 1, 0\}. \qquad (1)$$

There are no assembly costs, but one could easily treat the price p as the net unit revenue.

Two suppliers are capable of producing the required input. Each supplier's total cost of production is linear and it costs cq ($0 < c < c_S$) to produce q units. This cost is verifiable. We assume that both suppliers are risk averse and that they exhibit the same risk preferences. Hence orders are not driven by a cost heterogeneity or a difference in the risk aversion. A supplier's profits when he delivers x units and receives $w(x)$ are given by $\pi(w(x) - cs)$ where $\pi(.)$ is an increasing and concave function which captures risk aversion.

The suppliers are not reliable. For any given order, the quantity supplied is subject to uncertainty. Specifically, we consider that when he receives an order for q_i units supplier i ($i = 1,2$) delivers a quantity $q_i^D = \min\{q_i, s_i\}$ where s_i is the realization of a random variable $\tilde{s}_i \in [0, +\infty[$. The random variables $\tilde{s}_1$ and $\tilde{s}_2$ are independent. Specifically, we consider that $\tilde{s}_i$ follows an exponential distribution with a rate of occurrence λ_i. The cumulative distribution function is given by $F(s) = 1 - e^{-\lambda_i s}$ while the density is given by $f(t) = \lambda_i e^{-\lambda_i s}$.

The use of an exponential distribution allows us to capture the concept of reliability via the rate of occurrence. Notice that, given this distribution, the expectation of $\tilde{s}_i$ is equal to $\left(1/\lambda_i\right)$. Therefore, the ratio $\left(1/\lambda_i\right)$ can be understood as supplier i's capacity and serves as a proxy for his reliability. The larger λ_i is, the less reliable the supplier is.

We consider that supplier i's rate of occurrence, λ_i, depends (negatively) on some investment undertaken by the supplier. Specifically, let $I_i \equiv I(\lambda_i)$ denote the investment needed to achieve a rate of occurrence λ_i. The function $I(\lambda_i)$ is decreasing and convex so that $I'(\lambda_i) < 0$ and $I''(\lambda_i) \geq 0$, where we use prime to denote the first derivative and double prime the second derivative.

Notice finally that the probability of completing an order of size q, which is given by $1 - F(q) = e^{-\lambda q}$ is decreasing with the order size and with the rate of occurrence. Larger orders are therefore more likely to be incomplete.

We analyze two situations.

Situation 1: The manufacturer requires that suppliers invest an amount I_i ($i = 1,2$) which is verifiable. Given the suppliers' reliability he issues the orders and sets the suppliers' remuneration which must compensate each supplier for the investment undertaken initially.

Situation 2: The manufacturer issues a contract to each supplier stipulating the order size and the remuneration. He also suggests the level of investment each supplier should undertake but cannot verify the amount invested. If the contracts are accepted, the suppliers decide (individually and non-cooperatively) how much to invest.

In either situation, each supplier can accept or reject the contract. If a supplier rejects the offer we assume that he gets zero profits.

Clearly, situation 2 introduces some moral hazard since the manufacturer can no longer verify the investment decision and the resulting reliability of each supplier.

Before we start the analysis, we introduce the following notation and terminology.

Notation: Let $c_H = p + c_S + c_D$ and $c_L = p - c + c_S$ and let

$$\hat{c} = \frac{p + c_S + c_D}{p + c_S - c} = \frac{c_H}{c_L} > 1.$$

The variable $\hat{c}$ is the inverse of the standard newsvendor critical fractal.

Terminology: We refer to *order inflation* as a situation where the manufacturer orders more inputs than what he needs to meet demand. In this paper it occurs when he orders more than one unit of inputs. (Note that Tang et al. (2014) use the same terminology.)

4. Optimal contracts under verifiable investments.

In this situation, the manufacturer specifies the level of investments, the orders and monetary transfers that maximizes his expected profits. The only constraints are the participation constraints (one for each supplier) highlighting the condition under which the contract is accepted.

We solve for the optimal contract characterizing first the optimal transfers and orders for all possible reliability parameters and then the optimal investments. In the next subsection the parameters λ_1 and λ_2 will be considered as exogenous. This approach is very common, see for instance, Macho-Stadler and Pérez-Castrillo 2001.

2.1 Optimal monetary transfers and order sizes.

Supplier i accepts his contract provided

$$\int_0^{+\infty} \pi(w(q_i^D) - cq_i^D)f_i(s_i)ds_i - I_i \geq 0,$$

where $q_i^D = \min\{q_i, s_i\}$. This inequality can also be written as

$$\int_0^{q_i} \pi(w(s_i) - cs_i)\,f_i(s_i)dt_i + \int_{q_i}^{+\infty} \pi(w(q_i) - cq_i)f_i(s_i)dt_i - I_i \geq 0 \ (i = 1,2). \quad (2)$$

The first term measures the profits when the supplier does not complete the order, while the second term denotes the profits when the he does.

Lemma 1: *When investments are contractible, the optimal contracts are efficient and such that suppliers get no rents. Specifically, we have $w(q_i^D) = cq_i^D + \pi^{-1}(I_i)$ $(i = 1,2)$ so that suppliers are fully insured, meaning that their profits are the same whether or not they complete the order.*

 Proof: See Appendix 1.

Given the optimal transfers, we can re-write the manufacturer's profits as a function of the orders and the investments. The specific form depends on whether the

manufacturer uses order inflation, defined as setting total orders greater than one, or not. (See supplementary material for more information.)

- When the manufacturer does not rely on order inflation, so that $(q_1 + q_2) \leq 1$, he never has to discard any inputs and we have

$$\Pi(q_1, q_2, \lambda_1, \lambda_2)|_{q_1+q_2\leq 1} = (p - c)(q_1 + q_2) - c_S(1 - q_1 - q_2)$$

$$-c_L \left[\int_0^{q_1} F_1(s)ds + \int_0^{q_2} F_2(s)ds \right] - \sum_{i=1,2} \pi^{-1}(I_i). \tag{3}$$

The monetary transfer covers the cost of production and the investment. The last term in (3) measures the compensation for the investments. The first term reflects the profits when both orders are completed. The second term accounts for the cost of not meeting the demand when orders are strictly below one. Finally the third term reflects the losses associated with potential order incompletion.

- When the manufacturer relies on order inflation, that is when $(q_1 + q_2) \geq 1$, he may have to discard some of the inputs and we have

$$\Pi(q_1, q_2, \lambda_1, \lambda_2)|_{q_1+q_2\geq 1} = p - c(q_1 + q_2) - c_D(q_1 + q_2 - 1)$$

$$-c_H \left[\int_0^{1-q_1} F_2(s)ds + \int_0^{1-q_2} F_1(s)ds + \int_{1-q_2}^{q_1} F_2(1 - s)F_1(s)ds \right] \tag{4}$$

$$+(c + c_D) \left[\int_0^{q_1} F_1(s)ds + \int_0^{q_2} F_2(s)ds \right] - \sum_{i=1,2} \pi^{-1}(I_i).$$

The last term is once again the compensation for the investments. Recall that the manufacturer commits to pay the cost of the quantity delivered. When more than one unit is ordered and delivered, the manufacturer must pay for the cost of producing inputs he will not use. Therefore, the first two terms measures the profits upon order completion, taking into account the fact that he sells at most one unit. From these profits one must withdraw the cost of disposing of unwanted inputs when more than one unit is delivered (third term). The fourth term measures the cost of not being able to satisfy the demand when the sum of deliveries is less than one. Finally the fifth term measures the savings associated with individual order incompletions which means that less inputs have been produced and less inputs must be disposed of.

It is straightforward to verify that the profits function is continuous at $(q_1 + q_2) = 1$ and we have

$$\Pi(q_1, q_2, \lambda_1, \lambda_2)\big|_{q_1+q_2=1}$$

$$= (p - c) - c_L \left[\int_0^{q_1} F_1(s)ds + \int_0^{q_2} F_2(s)ds \right] - \sum_{i=1,2} \pi^{-1}(I_i). \tag{5}$$

Lemma 2 establishes two intuitive results.

Lemma 2: *The manufacturer always orders at least one unit of inputs. If at least one of the supplier is reliable then he orders exactly one unit of inputs. More specifically, when supplier i ($i = 1$ or 2) is reliable, meaning that $\lambda_i = 0$ while supplier j ($j = 1$ or $2, j \neq i$) is not meaning that $\lambda_j > 0$, we have $q_i = 1$ and $q_j = 0$. When both suppliers are reliable ($\lambda_i = 0$ for $i = 1,2$) then any combination of orders such that $(q_1 + q_2) = 1$ is optimal.*

Proof: For any given rates of occurrences, we have

$$\frac{\partial}{\partial q_i} \Pi(q_1, q_2, \lambda_1, \lambda_2)\big|_{q_1+q_2 \leq 1} = c_L(1 - F_i(q_i)) \geq 0. \tag{6}$$

Therefore the optimal orders are such that $(q_1 + q_2) \geq 1$.

For all $q_i \geq 1 - q_j$ with $i, j = 1,2$ and $i \neq j$, we have

$$\frac{\partial \Pi(q_1, q_2, \lambda_1, \lambda_2)}{\partial q_i} = [1 - F_i(q_i)][F_j(1 - q_i)c_H - (c_H - c_L)]. \tag{7}$$

If supplier j ($j = 1,2$) has a zero rate of occurrence then $F_j(1 - q_i) = 0$ and the expected profits are decreasing with q_i so that it is optimal to set $q_i = 0$ and $q_j = 1$. When both suppliers are reliable, the function $\Pi(q_1, q_2, \lambda_1, \lambda_2)$ decreases in q_1 and q_2 for all $q_1 + q_2 \geq 1$ and, according to (6), increases in q_1 and q_2 for all $q_1 + q_2 \leq 1$. It reaches a maximum along the line $q_1 + q_2 = 1$.∎

We now characterize the optimal orders when both suppliers are unreliable ($\lambda_i > 0$ for $i = 1,2$). To do so, we analyze the equivalent of the "best-reply functions" that is the optimal order to supplier i as a function to the supplier j's order.

Using (7) one can easily show that is optimal for the manufacturer to rely on order inflation and set $q_i > 1 - q_j$ provided

$$\frac{\partial \Pi(q_1, q_2, \lambda_1, \lambda_2)}{\partial q_i}\bigg|_{q_i=1-q_j} > 0 \Leftrightarrow \lambda_j q_j > \ln \hat{c}. \tag{8}$$

Recall that $\hat{c} = {}^{c_H}/_{c_L} > 1$. When (8) holds, the optimal order for supplier i is such that $F_j(1 - q_i)c_H - (c_H - c_L) = 0$. It follows that the best reply function $q_i(q_j)$ ($i, j = 1,2$ $i \neq j$) is given by

$$q_i(q_j) = \begin{cases} 1 - q_j \text{ for } q_j \leq \min\left\{1, \dfrac{\ln \hat{c}}{\lambda_j}\right\}, \\[4mm] 1 - \dfrac{\ln \hat{c}}{\lambda_j} \text{ for } q_j \geq \min\left\{1, \dfrac{\ln \hat{c}}{\lambda_j}\right\}. \end{cases} \tag{9}$$

Graphs 1 and 2 below represent the two possible outcomes. These show that there may be a unique equilibrium in which the manufacturer relies on order inflation (Graph 1). This equilibrium is more likely to arise when suppliers are particularly unreliable. Alternatively, there may be a multiplicity of equilibria such that the manufacturer orders no more than one unit of inputs (Graph 2). Our findings are summarized in proposition 1 below the graphs.
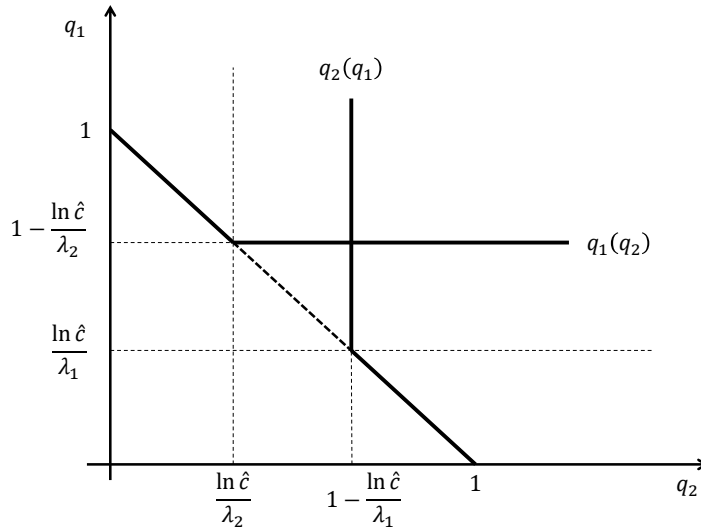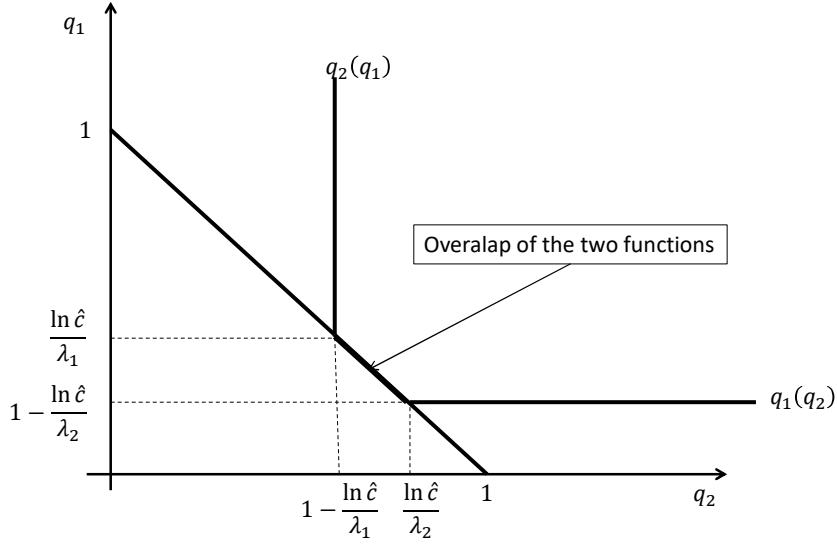

Graph 1: Best reply functions for the case $\dfrac{\ln \hat{c}}{\lambda_i} < 1 - \dfrac{\ln \hat{c}}{\lambda_j}$ $i, j = 1,2 \; i \neq j$.

In the situation above, the optimal orders are such that

$$q_i = 1 - \frac{\ln \hat{c}}{\lambda_j} \; i, j = 1,2 \text{ and } i \neq j.$$

If we now allow λ_j to fall to such an extent that we have $\dfrac{\ln \hat{c}}{\lambda_i} > 1 - \dfrac{\ln \hat{c}}{\lambda_j}$ then the best reply functions are given in the graph below.

Graph 2: Best reply functions for the case $\frac{\ln \hat{c}}{\lambda_i} > 1 - \frac{\ln \hat{c}}{\lambda_j}$ $i, j = 1,2$ $i \neq j$.

In this case we have a multiplicity of equilibria characterized as follows:

$$q_1 + q_2 = 1 \ and \ max\left\{1 - \frac{\ln \hat{c}}{\lambda_j}, 0\right\} \leq q_i \leq min\left\{\frac{\ln \hat{c}}{\lambda_i}, 1\right\} \ with \ i, j = 1,2 \ i \neq j.$$

We can now state our result.

Proposition 1: *Consider the level curve characterized by*

$$\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} = \ln \hat{c} \, , where \ \hat{c} = \frac{p + c_S + c_D}{p + c_S - c}.$$

- *When $\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} \leq \ln \hat{c}$ we have a multiplicity of equilibria. All are such that the sum of orders equals 1 and*

$$max\left\{1 - \frac{\ln \hat{c}}{\lambda_j}, 0\right\} \leq q_i \leq min\left\{\frac{\ln \hat{c}}{\lambda_i}, 1\right\} \ with \ i, j = 1,2 \ i \neq j.$$

- *When $\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} \geq \ln \hat{c}$ the equilibrium is unique and such that the sum of orders is at least equal to 1 (meaning that there is potential order inflation) and we have*

$$q_i = 1 - \frac{\ln \hat{c}}{\lambda_j} \ i, j = 1,2 \ and \ i \neq j.$$

Proof: The proof follows from the analysis of the best reply functions. Notice that when there is order inflation we have $q_1 + q_2 = 2 - \ln \hat{c} \left(\frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2}\right)$. In the Appendix, we show that the second order condition holds whether or not there is order inflation.■
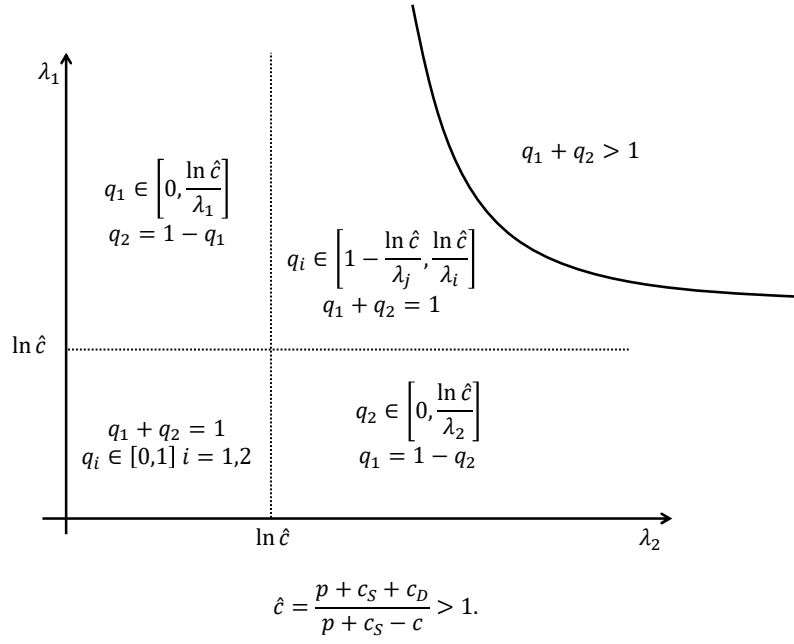
The graphs below give a visual representation of the optimal order. Graph 3 represents all (λ_1, λ_2) for which there is order inflation. Graph 4 depicts, with more precision, the multiple equilibria that occur when there is no order inflation.
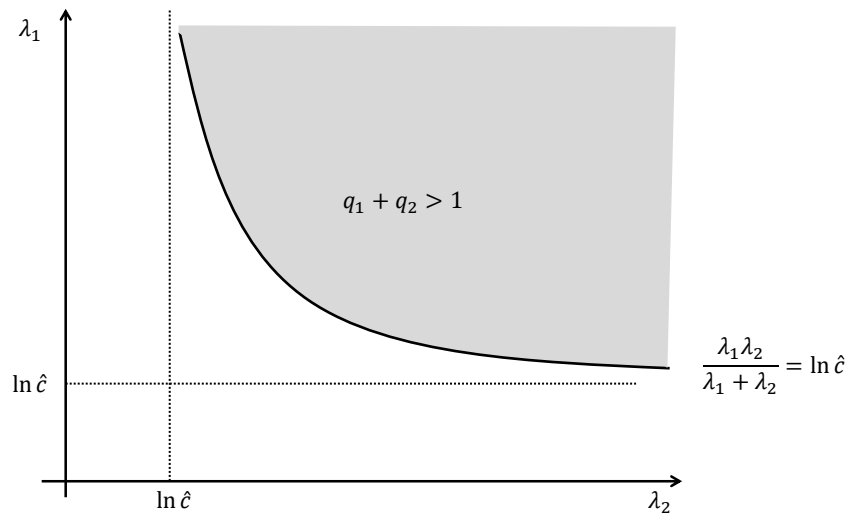
Graph 3: shows all (λ_1, λ_2) for which we have order inflation.

The equilibrium is unique above the level curve and we have order inflation. On the level curve the equilibrium is unique and we have $q_1 + q_2 = 1$ and $q_i = 1 - \frac{\ln \hat{c}}{\lambda_j}$ $i, j = 1, 2$ and $i \neq j$. Below the level curve we have a multiplicity of equilibria but the orders sum to 1. Graph 4, below, emphasizes those equilibria.

Clearly, whenever we have $\lambda_1 = \lambda_2$ only two of the four regions depicted above are relevant (the south west and north east ones) and one particular equilibrium is such that $q_1 = q_2 = \frac{1}{2}$.

The graph shows axes λ_1 (vertical) and λ_2 (horizontal), with $\ln \hat{c}$ marked on both axes.

Quadrant top-left:
$$q_1 \in \left[0, \frac{\ln \hat{c}}{\lambda_1}\right]$$
$$q_2 = 1 - q_1$$

Center region:
$$q_i \in \left[1 - \frac{\ln \hat{c}}{\lambda_j}, \frac{\ln \hat{c}}{\lambda_i}\right]$$
$$q_1 + q_2 = 1$$

Top-right:
$$q_1 + q_2 > 1$$

Bottom-left:
$$q_1 + q_2 = 1$$
$$q_i \in [0,1] \; i = 1,2$$

Bottom-right:
$$q_2 \in \left[0, \frac{\ln \hat{c}}{\lambda_2}\right]$$
$$q_1 = 1 - q_2$$

$$\hat{c} = \frac{p + c_S + c_D}{p + c_S - c} > 1.$$

Graph 4: Optimal orders when $q_1 + q_2 = 1$.

Before we characterize optimal investments, we present some comparative statics.

Lemma 3: *Order inflation is optimal for a **wider** range of parameters when the price of the manufacturer's output increases or when the cost associated with a shortage of input increases. By opposition, order inflation is optimal for a **narrower** range of parameters when the suppliers' marginal cost increases or when the cost associated with an excess of inputs increases.*

The results stated above are very intuitive and driven from the fact that

$$\frac{\partial \hat{c}}{\partial p} < 0 \text{ and } \frac{\partial \hat{c}}{\partial c_S} < 0 \text{ and that } \frac{\partial \hat{c}}{\partial c} > 0 \text{ and } \frac{\partial \hat{c}}{\partial c_D} > 0.$$

2.2 Optimal investments.

Taking into account the optimal orders, we now solve for the optimal investments. One can approach this problem as characterizing the optimal rates of occurrence. We then compare the marginal revenue to the marginal cost associated with an increase in reliability.

Clearly we expect that a marginal increase in the rate of occurrence will have a negative impact on the revenue.

Lemma 4: *Whether or not the manufacturer relies on order inflation, the marginal impact on the revenue associated with an increase in λ_i is negative and increasing in λ_i. In other*
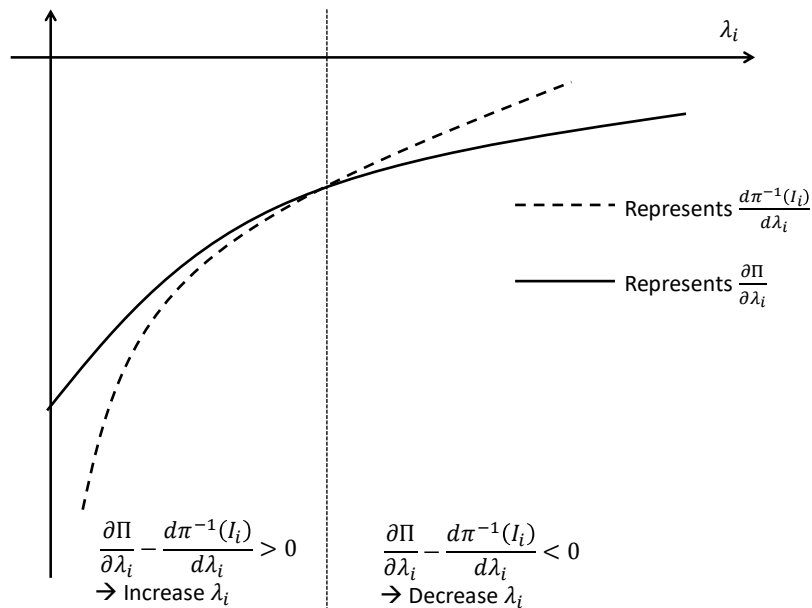
words, the manufacturer's revenue is decreasing and convex in λ_i ($i = 1,2$). It is also continuously differentiable in λ_i ($i = 1,2$).

Proof*: See Appendix.*

What Lemma 4 states is that increasing the rate of occurrence of a supplier impacts the revenue negatively (as expected) but the marginal impact decreases as the supplier becomes more and more unreliable.

The cost associated with the investments is given by $\sum_{i=1,2} \pi^{-1}(I_i)$. Thus the marginal cost is negative and increasing since we consider that the function $I(\lambda_i)$ is decreasing and convex while $\pi(.)$ is increasing and concave.

Depending on the shape of the investment function, several possibilities arise. To guarantee the existence of an interior solution it is sufficient to impose that the function $\pi^{-1}(I_i)$ be everywhere more convex than $\Pi(q_1, q_2, \lambda_1, \lambda_2)$ as shown in graph 5 below. It represents the marginal cost and marginal loss associated with an increase in λ_i.

$$\frac{\partial \Pi}{\partial \lambda_i} - \frac{d\pi^{-1}(I_i)}{d\lambda_i} > 0$$
→ Increase λ_i

$$\frac{\partial \Pi}{\partial \lambda_i} - \frac{d\pi^{-1}(I_i)}{d\lambda_i} < 0$$
→ Decrease λ_i

Legend:
- Represents $\frac{d\pi^{-1}(I_i)}{d\lambda_i}$
- Represents $\frac{\partial \Pi}{\partial \lambda_i}$

Graph 5: Existence of an interior solution.

Assuming that an interior solution does exist, we can characterize it.

Proposition 2: *Provided the investment function is sufficiently convex, there exists a unique interior solution for the occurrence rates. A symmetric solution (λ^*, q^*) is characterized as follows.*

- *Assume that there exists $\lambda^* \leq 2\ln\hat{c}$ which solves*

$$\frac{d\pi^{-1}\big(I(\lambda^*)\big)}{d\lambda^*} = -c_L \int_0^{\frac{1}{2}} t[1 - F_i(s)]ds,$$

then the optimal rate of occurrence is λ^* *and* $q^* = \frac{1}{2}.$

▪ *Assume that there exists* $\lambda^* \geq 2 \ln \hat{c}$ *which solves*

$$\frac{d\pi^{-1}\big(I(\lambda^*)\big)}{d\lambda^*} =$$

$$c_H \left[\int_{1-q^*}^{q^*} s[1 - F(s)][1 - F(1 - s)]ds - (1 - F(1 - q^*)) \int_0^{q^*} s[1 - F(s)]ds \right],$$

then the optimal rate of occurrence is λ^* *and* $q^* = 1 - \frac{\ln \hat{c}}{\lambda^*}.$

Proof: See the proof of Lemma 4 for the equation characterizing the optimal rate of occurrence. The rest of the proposition is then straightforward as it relies on information given in previous propositions and Lemmas. Notice that the symmetric equilibrium rate λ^* is on the level curve $\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} = \ln \hat{c}$ provided $\lambda^* = 2 \ln \hat{c}.$ ∎

Whether or not investments are low and there is order inflation in equilibrium depends on all exogenous parameters (price and costs) as well as on the investment function. Notice that as c_H increases (relative to c_L) so does $2 \ln \hat{c}$ and therefore it is more likely that the manufacturer increases his investment and issues symmetric orders equal to ½. This intuition is confirmed in what follows.

To get a better understanding of what type of equilibrium arises, let us consider that $\pi(w) = \sqrt{w}$ and that $I(\lambda) = e^{-\lambda/2}$. To have an interior solution assume that $c_L \in\ ]0,8]$ and of course we have $c_H \geq c_L$. (Please see Appendix 4 for greater details of the calculations as well as the reason why we must have $c_L \leq 8$.) Graph 6, below, shows the optimal contracting strategy for all possible, $c_L \in\ ]0,8]$ and $c_H \geq c_L$.

One possible interpretation of the graph above is that the manufacturer is more inclined to invest in the suppliers' reliability and then refrain from using order inflation when $c_H - c_L = c + c_D$ increases, that is when production costs and the cost of disposing of unwanted inputs are large.

This completes the analysis of the optimal sourcing strategy when investments are contractible.
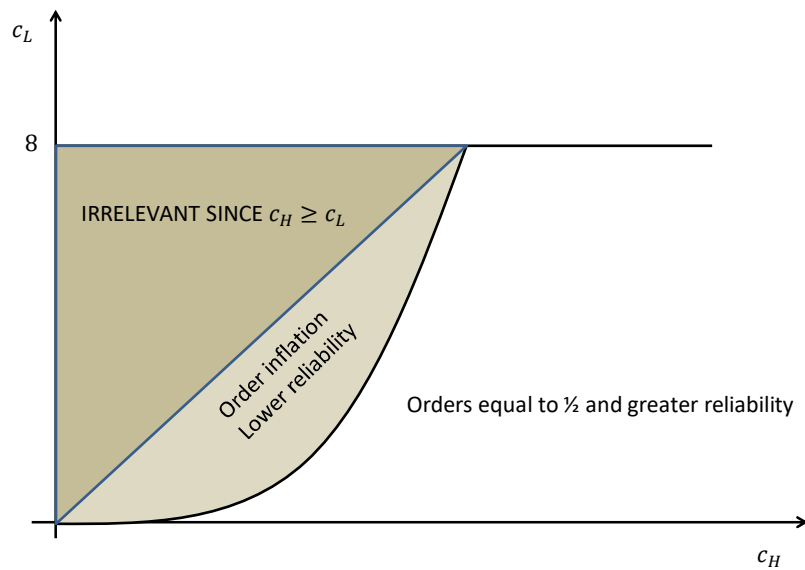
Graph 6: Summary of the manufacturer's optimal contracts.

5. Optimal contracts under moral hazard

We now consider a situation where the manufacturer cannot verify the investment undertaken by each of the suppliers. He can make some recommendation in terms of the reliability that he is expecting but cannot verify the investment undertaken by each supplier. Thus, the manufacturer must take into account, in addition to the participation constraints, an incentive constraint for each supplier when designing the optimal contracts. The questions we address in this section are: how does the presence of moral hazard affect the optimal contracts? In particular will we observe that the manufacturer relies more heavily on order inflation as he deals with less reliable suppliers?

Under moral hazard each contract specifies the order size and the monetary transfers to the supplier which depend on how much is delivered as well as the manufacturer's recommended reliability. We introduce, without loss of generalities, the reward functions $t^S(.)$ and $t^F(.)$ such that: $w(q_i) = cq_i + t^S(q_i, \hat{\lambda}_i)$ is the payment upon *successful* delivery and $w(s_i) = cs_i + t^F(q_i, \hat{\lambda}_i)$ is the transfer when the supplier *fails* to complete the order and delivers a quantity $s_i < q_i$. The variable $\hat{\lambda}_i$ is the *recommended* level of reliability.

The optimal contract must satisfy the participation constraints given by (2). Taking into account the expressions for the remuneration functions, (2) can be re-written as

$$\pi\left(t^S(q_i, \hat{\lambda}_i)\right) - \left[\pi\left(t^S(q_i, \hat{\lambda}_i)\right) - \pi\left(t^F(q_i, \hat{\lambda}_i)\right)\right] F(q_i) - I_i \geq 0 \quad (i = 1,2). \qquad (10)$$

The optimal contract must also satisfy an incentive constraint which states that the supplier's optimal level of reliability must match the *recommended* level. Thus the first order condition states that the derivative of the above with respect to λ_i must be equal to zero at $\lambda_i = \hat{\lambda}_i$. Notice that with the exponential distribution function, we have $\frac{\partial F_i(q_i)}{\partial \lambda_i} = q_i[1 - F_i(q_i)]$. Therefore, we can write the incentive constraints as

$$-q_i[1 - F_i(q_i)]|_{\hat{\lambda}_i} \left[\pi\left(t^S(q_i, \hat{\lambda}_i)\right) - \pi\left(t^F(q_i, \hat{\lambda}_i)\right)\right] - I_i'(\hat{\lambda}_i) = 0 \quad (i = 1,2). \qquad (11)$$

The second order condition guaranteeing that $\hat{\lambda}_i$ is a maximum, holds provided $I''(\hat{\lambda}_i) + q_i I'(\hat{\lambda}_i) > 0$, which holds provided the investment function sufficiently convex.

Lemma 5: *The optimal contract is such that the functions $t^S(q_i, \hat{\lambda}_i)$ and $t^F(q_i, \hat{\lambda}_i)$ depend on both, the order size and the recommended rate of occurrence. Specifically we have*

$$t^S(q_i, \hat{\lambda}_i) = \pi^{-1}\left(I_i(\hat{\lambda}_i) - I_i'(\hat{\lambda}_i) \frac{F_i(q_i)}{q_i(1 - F_i(q_i))}\right) \text{ and } t^F(q_i, \hat{\lambda}_i) = \pi^{-1}\left(I_i(\hat{\lambda}_i) + I_i'(\hat{\lambda}_i)\frac{1}{q_i}\right).$$

Proof: In equilibrium the optimal contract is such that both, the participation constraint and the incentive constraint bind. The above functions satisfy this requirement. ∎

As one would expect, the contract is no longer efficient and each supplier is penalized in the event of a delay and gets a bonus when the order is completed since

$$\left(I_i + I_i'\frac{1}{q_i}\right) < I_i < \left(I_i - I_i'\frac{F_i(q_i)}{q_i(1 - F_i(q_i))}\right). \qquad (12)$$

The implementation of penalties and bonuses introduces some dispersion which is inefficient but necessary to achieve incentive compatibility. However, and more interestingly, (11) implies that the payoff distortion imposed on the suppliers, and hence the extent to which the contract is inefficient, depends on the order size. This means that the order size plays an additional strategic role. Lemma 6, below, brings to light the relationship between investment, contract efficiency and order sizes.

Lemma 6: *Given any order set below the supplier's capacity (i.e. $q_i < {}^1/_{\lambda_i}$) any given investments can be implemented via less profit dispersion by marginally increasing the*

order size. Given any order set above the supplier's capacity (i.e. $q_i > {}^1\!/_{\lambda_i}$) any given investments can be implemented via less profit dispersion by marginally decreasing the order size.

Proof: Notice that the incentive constraint requires that

$$\left[\pi\left(t^S(q_i, \hat{\lambda}_i)\right) - \pi\left(t^F(q_i, \hat{\lambda}_i)\right)\right] = \frac{-I_i'(\hat{\lambda}_i)}{q_i[1 - F_i(q_i)]|_{\hat{\lambda}_i}}. \tag{13}$$

Simple calculations lead us to

$$\frac{dq_i[1 - F_i(q_i)]}{dq_i} = \lambda_i[1 - F_i(q_i)]\left[\frac{1}{\lambda_i} - q_i\right]. \tag{14}$$

Therefore, for any given investment, the required inefficiency decreases with order size provided $\left[\frac{1}{\lambda_i} - q_i\right] > 0.\blacksquare$

From Lemma 6 we learn that the optimal contract under moral hazard relies on fine-tuning the order size and the payoff distortion:

- For any given order size, the greater the gap between what the supplier gets when he completes the contract and what he gets when he fails, the greater the incentive to invest as indicated by (11).
- When the order is small (below capacity), increasing its size has a positive impact on the level of investment for any given rent distortion. Low orders are more likely to be completed. Therefore the supplier is more likely to be rewarded and has a stronger incentive to invest. Under such circumstances, the manufacturer can increase the order slightly to incentivise the supplier without having to rely on a greater discrepancy between transfers.
- A large order is, by opposition, more likely to be unfulfilled so that the supplier is more likely to be penalized when he receives a large order. Thus, reducing its size has a positive impact on the investment.

Clearly, the introduction of moral hazard is costly and does not allow the manufacturer to reduce his reliance on order inflation and reach higher investments. It can only be one or the other or none of these. But, it is not clear that he will always have to increase his reliance on order inflation and face less reliable suppliers.

Using Lemma 5, one can easily verify that the manufacturer's profits under moral hazard can be written

$$\widehat{\Pi}(q_1, q_2, \lambda_1, \lambda_2) = \Pi(q_1, q_2, \lambda_1, \lambda_2) + \sum_{i=1,2} \pi^{-1}(I_i)$$

$$- \sum_{i=1,2} \left[F(q_i)\pi^{-1}\left(I_i - I_i'\frac{1}{q_i}\right) + (1 - F(q_i))\pi^{-1}\left(I_i - I_i'\frac{F(q_i)}{q_i(1 - F(q_i))}\right) \right],$$

(15)

Where $\Pi(q_1, q_2, \lambda_1, \lambda_2)$ is given by (3), (4) or (5) depending on whether the sum of orders is below, above or equal to one. The introduction of moral hazard increases the cost towards covering the investments and the manufacturer's profits decrease.

In order to analyse how the manufacturer optimally manages outsourcing in the presence of moral hazard we consider once again the specific form $\pi(w) = \sqrt{w}$. In this case we have

$$\widehat{\Pi}(q_1, q_2, \lambda_1, \lambda_2) = \Pi(q_1, q_2, \lambda_1, \lambda_2)$$

$$+ \sum_{i=1,2} I_i^2 - \sum_{i=1,2} \left[\frac{F(q_i)}{(1 - F(q_i))} \frac{(I_i')^2}{(q_i)^2} \right].$$

(16)

Assume once again that $I(\lambda) = e^{-\lambda/2}$. Using Proposition 2 we are able to fully characterize the symmetric information solution. We then evaluate the derivative of $\widehat{\Pi}(q_1, q_2, \lambda_1, \lambda_2)$ at the symmetric information solution.

Consider the case where the exogenous parameters are such that investments are low and the manufacturer relies on order inflation under symmetric information. In such a case, investments are less of a priority (since they are low) and the manufacturer may be able to reduce the cost of implementing these investments (or lesser ones) under moral hazard by reducing the order sizes. This scenario is illustrated in the following table. Assume $c_L = 1$.

	$c_H = 2$	$c_H = 4$
Symmetric Information Solution	$\lambda^* = 3.06$ $q^* = 0.77$	$\lambda^* = 3.15$ $q^* = 0.78$
Impact of Moral Hazard	**Reduced** order size and reduced reliability	

Under symmetric information manufacturer sets orders above the suppliers' capacity $(q^* > 1/\lambda^*)$ and sets low investments. To minimize the cost of moral hazard he implements lower investments and reduces the order size, which, following Lemma 6, reduces the cost of moral hazard.

Consider now the case where the exogenous parameters are such that investments are high and the manufacturer sets orders equal to ½ under symmetric information. In such a case, the manufacturer may be able to implement these investments or greater ones at a lower cost by increasing the order sizes. This scenario is illustrated in the following table. Assume $c_L = 6$.

	$c_H \geq 8$
Symmetric Information Solution	$\lambda^* = 0.43$ $q^* = 0.5$
Impact of Moral Hazard	**Increased** order size and **increased** reliability

Under symmetric information manufacturer sets orders largely below the suppliers' capacity $(q^* < 1/\lambda^*)$ and sets high investments. Under moral hazard he implements larger investments via a greater reliance on order inflation.

Therefore we conclude that the manufacturer will increase his reliance on order inflation unless the orders under symmetric information are high (first table). This enables him to alleviate the cost triggered by the incentive constraint.

6. Conclusions

There are several approaches to managing supply disruptions. Here we consider the situation of a manufacturer who may lack the ability to diversify his supply base and uses order inflation and/or investments in process reliability when contracting two risk averse suppliers. We consider that these investments can be subject to moral hazard. We rely on a random capacity model of disruption. In such a model, the order size does not affect the average production but impacts the probability of disruption.

When investments are verifiable we show that the manufacturer is more inclined to invest in the suppliers' reliability and then refrain from using order inflation when the suppliers' production costs and the cost of disposing of unwanted inputs are large.

When investments are not verifiable we show that the order sizes can be used strategically as incentive devises. An important conclusion is that the manufacturer does not always increase his reliance on order inflation and face less reliable suppliers. In some instances he induces suppliers to undertake larger investments in reliability by increasing the order size. In other instances he is able to reduce his reliance on order inflation.

The analysis provided here can be extended in many directions. Wang et al. 2014 investigate investment in process improvement in a context where suppliers sell their output to several manufacturers. They analyse the manufacturers' incentive to invest in process improvement in the presence of externalities.

APPENDIX

Appendix 1: Proof or Lemma 1

Let $R^S(x_1, x_2)$ denote the manufacturer's revenue in the event of a shortage of inputs and $R^D(x_1, x_2)$ denote the manufacturer's revenue when he receives an excessive amount of inputs:

$$R^S(x_1, x_2) = p(x_1 + x_2) - c_S(1 - x_1 - x_2) \text{ and } R^D(x_1, x_2) = p - c_D(x_1 + x_2 - 1).$$

Finally let $W(x_1, x_2)$ denote the sum of transfers to the suppliers:

$$W(x_1, x_2) = w(x_1) + w(x_2).$$

Assume the orders submitted are such that $q_1 + q_2 \leq 1$. In this case, the manufacturer's expected revenue is given by the following expression:

$$\Pi(q_1, q_2, \lambda_1, \lambda_2) = \Pi_{s_1 \leq q_1} + \Pi_{s_1 \geq q_1}$$

where

$$\Pi_{s_1 \leq q_1} = \int_0^{q_1} \left[\int_0^{q_2} \left(R^S(s_1, s_2) - W(s_1, s_2) \right) f_2(s_2) ds_2 \right. $$
$$\left. + \int_{q_2}^{\infty} \left(R^S(s_1, q_2) - W(s_1, q_2) \right) f_2(s_2) ds_2 \right] f_1(s_1) ds_1$$

and

$$\Pi_{s_1 \geq q_1} = \int_{q_1}^{\infty} \left[\int_0^{q_2} (R^S(q_1, s_2) - W(q_1, s_2)) f_2(s_2) ds_2 \right.$$

$$\left. + \int_{q_2}^{\infty} (R^S(q_1, q_2) - W(q_1, q_2)) f_2(s_2) ds_2 \right] f_1(s_1) ds_1$$

Assume the orders submitted are such that $q_1 + q_2 \geq 1$. In this case, the manufacturer's expected revenue is given by the following expression

$$\Pi(q_1, q_2, \lambda_1, \lambda_2) = \Pi_{s_1 \leq 1 - q_2} + \Pi_{s_1 \in [1 - q_2, q_1]} + \Pi_{s_1 \geq q_1}$$

where

$$\Pi_{s_1 \leq 1 - q_2} = \int_0^{1 - q_2} \left[\int_0^{q_2} (R^S(s_1, s_2) - W(s_1, s_2)) f_2(s_2) ds_2 \right.$$

$$\left. + \int_{q_2}^{\infty} (R^S(s_1, q_2) - W(s_1, q_2)) f_2(s_2) ds_2 \right] f_1(s_1) ds_1$$

and

$$\Pi_{s_1 \in [1 - q_2, q_1]} = \int_{1 - q_2}^{q_1} \left[\int_0^{1 - t_1} (R^S(s_1, s_2) - W(s_1, s_2)) f_2(s_2) ds_2 \right.$$

$$+ \int_{1 - t_1}^{q_2} (R^D(s_1, s_2) - W(s_1, s_2)) f_2(s_2) ds_2$$

$$\left. + \int_{q_2}^{\infty} (R^D(s_1, q_2) - W(s_1, q_2)) f_2(s_2) dt_2 \right] f_1(s_1) ds_1$$

and finally:

$$\Pi_{s_1 \geq q_1} = \int_{q_1}^{\infty} \left[\int_0^{1 - q_1} (R^S(q_1, s_2) - W(q_1, s_2)) f_2(s_2) ds_2 \right.$$

$$+ \int_{1 - q_1}^{q_2} (R^D(q_1 s_2) - W(q_1, s_2)) f_2(s_2) ds_2$$

$$\left. + \int_{q_2}^{\infty} (R^D(q_1, q_2) - W(q_1, q_2)) f_2(s_2) ds_2 \right] f_1(s_1) ds_1.$$

The Lagrangian, which takes into account constraint (2) in the text, can be written as

$$\mathcal{L} = \Pi(q_1, q_2, \lambda_1, \lambda_2)$$

$$- \sum_{i=1,2} \beta_i \left[\int_0^{q_i} \pi(w(s_i) - cs_i) f_i(s_i) ds_i + \int_{q_i}^{+\infty} \pi(w(q_i) - cq_i) f_i(s_i) ds_i - I_i \right],$$

where β_i $(i = 1,2)$ are the Lagrangian multipliers and $I_i \equiv I(\lambda_i)$ for $i = 1,2$.

The following first order conditions must hold for $i = 1,2$ and any $x_i \in \{t_i, q_i\}$

$$\beta_i \pi'(w(x_i) - cx_i) - 1 = 0 \ (i = 1,2).$$

It follows that $w(x_i) - cx_i$ is independent of how much is produced. To complete the proof of the Lemma one must use the fact that the participation constraint must hold.

Appendix 2: Proof of Second Order Condition for Proposition 1

Recall that for all $q_i \geq 1 - q_j$ with $i,j = 1,2$ and $i \neq j$, the first order condition is given by

$$\frac{\partial \Pi(q_1, q_2, \lambda_1, \lambda_2)}{\partial q_i} = [1 - F_i(q_i)][F_j(1 - q_i)c_H - (c_H - c_L)]. \qquad (17)$$

Assume that the solution is unique and interior so that the second term is equal to zero.

In such a case the Hessian matrix is given by (at the solution)

$$H = c_H[1 - F_i(q_i)] \begin{bmatrix} -f_2(1 - q_1) & 0 \\ 0 & -f_1(1 - q_2) \end{bmatrix}.$$

The above is clearly negative definite.

Assume that we have a multiplicity of solutions and for each of these, the optimal orders are such that the second term of (17) is negative at $q_i = 1 - q_j$. Since the second term is decreasing in q_i, it is negative for all $q_i > 1 - q_j$ and therefore $\Pi(q_1, q_2, \lambda_1, \lambda_2)$ is decreasing and maximized at $q_i = 1 - q_j$.

Appendix 3: Proof of Lemma 3.

Consider graph 3 in the text. First, consider all (λ_1, λ_2) located strictly below the level curve for which there is no order inflation and for which there is a multiplicity of equilibria such that $q_1 + q_2 = 1$. Given the multiplicity of equilibria, there is no loss in generalities from assuming that the manufacturer selects orders located in the middle of the non-empty and non-singleton interval over which the best reply functions overlap (see graph 2 in the text). In such a situation, the manufacturer's revenue is given by (5) in the text and a marginal increase in the rates of occurrence has no impact on the individual orders. It follows that

$$\frac{\partial}{\partial \lambda_i} \Pi(q_1, q_2, \lambda_1, \lambda_2)|_{\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} < \ln \hat{c}} = -c_L \int_0^{q_i} \frac{\partial F_i(s)}{\partial \lambda_i} ds < 0$$

since $\frac{\partial F_i(s)}{\partial \lambda_i} = s[1 - F_i(s)] \geq 0$. Finally we have

$$\frac{\partial^2}{\partial \lambda_i^2} \Pi(q_1, q_2, \lambda_1, \lambda_2)|_{\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} < \ln \hat{c}} = -c_L \int_0^{q_i} \frac{\partial^2 F_i(s)}{\partial \lambda_i^2} ds > 0$$

since $\frac{\partial^2 F_i(s)}{\partial \lambda_i^2} = -s^2[1 - F_i(s)] \leq 0$.

Now let us consider all (λ_1, λ_2) located on or above the level curve for which we have an interior solution. In such a situation, the manufacturer's revenue is given by (P2) in the text and the optimal orders (even on the level curve) solve $\frac{\partial \Pi(q_1, q_2, \lambda_1, \lambda_2)}{\partial q_i} = 0$ for $i = 1,2$. Using the fact that the first order condition holds in relation to the orders we have

$$\frac{\partial}{\partial \lambda_i} \Pi(q_1, q_2, \lambda_1, \lambda_2)|_{\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} \geq \ln \hat{c}} = (c_H - c_L) \int_0^{q_i} \frac{\partial F_i(s)}{\partial \lambda_i} ds$$

$$-c_H \left[\int_0^{1-q_j} \frac{\partial F_i(s)}{\partial \lambda_i} ds + \int_{1-q_j}^{q_i} \frac{\partial F_i(s)}{\partial \lambda_i} F_j(1-s) ds \right]$$

Given that $\frac{\partial \Pi(q_1, q_2, \lambda_1, \lambda_2)}{\partial q_i} = 0$, and using (17), we can replace $(c_H - c_L)$ by $F_j(1 - q_i)c_H$ and since $\frac{\partial F_i(s)}{\partial \lambda_i} = s[1 - F_i(s)]$ we can re-write the right hand side of the above as:

$$c_H \left[\int_{1-q_j}^{q_i} s[1 - F_i(s)][1 - F_j(1-s)] ds - (1 - F_j(1 - q_i)) \int_0^{q_i} s[1 - F_i(s)] ds \right].$$

Notice first of all that for all (λ_1, λ_2) located on the level curve the term

$$\int_{1-q_j}^{q_i} s[1 - F_i(s)][1 - F_j(1-s)] ds = 0$$

since $q_i = 1 - q_j$. Thus $\frac{\partial}{\partial \lambda_i} \Pi(q_1, q_2, \lambda_1, \lambda_2)|_{\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} \geq \ln \hat{c}}$ is negative.

For all (λ_1, λ_2) located above the level curve notice that for any $t \leq q_i$ we have $[1 - F_j(1-s)] \leq [1 - F_j(1 - q_i)]$, therefore

$$\int_{1-q_j}^{q_i} s[1 - F_i(s)][1 - F_j(1 - s)]ds \leq [1 - F_j(1 - q_i)] \int_{1-q_j}^{q_i} s[1 - F_i(s)]ds.$$

It follows that $\frac{\partial}{\partial \lambda_i} \Pi(q_1, q_2, \lambda_1, \lambda_2)|_{\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} \geq \ln \hat{c}}$ is necessarily non-positive.

Finally we have

$$\frac{\partial^2}{\partial \lambda_i^2} \Pi(q_1, q_2, \lambda_1, \lambda_2)|_{\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} \geq \ln \hat{c}}$$

$$= c_H \left[(1 - F_j(1 - q_i)) \int_0^{q_i} s^2[1 - F_i(s)]ds \right.$$

$$\left. - \int_{1-q_j}^{q_i} s^2[1 - F_i(s)][1 - F_j(1 - s)]ds \right].$$

Using once more the fact that for any $s \leq q_i$ we have $[1 - F_j(1 - s)] \leq [1 - F_j(1 - q_i)]$ proves that the above expression is positive.

Finally we show that the function $\Pi(q_1, q_2, \lambda_1, \lambda_2)$ is continuously differentiable.

Just above the level curve $\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} = \ln \hat{c}$ we have

$$\lim_{\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} \to \ln \hat{c}} \frac{\partial}{\partial \lambda_i} \Pi(q_1, q_2, \lambda_1, \lambda_2)|_{\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} \geq \ln \hat{c}} = - c_H \left(1 - F_j(1 - q_i) \right) \int_0^{q_i} s[1 - F_i(s)]ds,$$

because $q_i = 1 - q_j$. The expression above is equal to $\frac{\partial}{\partial \lambda_i} \Pi(q_1, q_2, \lambda_1, \lambda_2)|_{\frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} < \ln \hat{c}}$ given above provided

$$c_H \left(1 - F_j(1 - q_i) \right) = c_L.$$

Since at the solution we have $F_j(1 - q_i)c_H = (c_H - c_L)$, the above is true.

Appendix 4: According to the conditions in Proposition 2, the manufacturer will submit orders equal to ½ and implement $\lambda^* \leq 2 \ln \hat{c}$ provided λ^* solves

$$-e^{-\lambda^*} = -\frac{c_L}{2(\lambda^*)^2} [2 - e^{-(\lambda^*/2)}(\lambda^* + 2)].$$

The manufacturer will submit orders $q^* = 1 - \frac{\ln \hat{c}}{\lambda^*}$ and implement $\lambda^* \geq 2 \ln \hat{c}$ provided λ^* solves

$$-e^{-\lambda^*} = c_H \frac{e^{-\lambda^*}}{2} \left[1 + \frac{2}{(\lambda^*)^2} (\lambda^* + 1)(1 - \ln \hat{c}) \right] - \frac{c_L}{(\lambda^*)^2}.$$

Evaluating either one of the two expressions above at $\lambda^* = 2 \ln \hat{c}$, one can show that setting $\lambda^* = 2 \ln \hat{c}$ is optimal for all c_H and c_L solving

$$4(\ln \hat{c})^2 = c_L \hat{c}^2 - c_H(1 + \ln \hat{c}).$$

(Recall that $\hat{c} = \frac{c_H}{c_L}$.) Before we represent the values of c_H and c_L for which there is order inflation notice that there does not exist an interior solution when $c_L \geq 8$ since we have

$$\lim_{\lambda^* \to 0} \frac{c_L}{2(\lambda^*)^2} \left[2 - e^{-(\lambda^*/2)}(\lambda^* + 2) \right] = -\frac{c_L}{8}.$$

One can indeed show that the Taylor expansion of the function $\frac{1}{(\lambda^*)^2} \left[2 - e^{-(\lambda^*/2)}(\lambda^* + 2) \right]$ is given by $\frac{1}{4} - \frac{x}{12} + \frac{x^2}{64} - \frac{x^3}{480} + \frac{x^4}{4608} + O(x^5)$ all the terms following ¼ converge to 0.

To have an interior solution we need to have

$$\lim_{\lambda^* \to 0} \frac{c_L}{2(\lambda^*)^2} \left[2 - e^{-(\lambda^*/2)}(\lambda^* + 2) \right] = \frac{c_L}{8} > -1.$$

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