UCD CENTRE FOR ECONOMIC RESEARCH

WORKING PAPER SERIES

2010

Distorted Trade Barriers: A Comment on "Distorted Gravity"

Matthew T Cole, University College Dublin

WP10/19

June 2010

UCD SCHOOL OF ECONOMICS
UNIVERSITY COLLEGE DUBLIN
BELFIELD DUBLIN 4

Distorted Trade Barriers: A Comment on "Distorted Gravity"

Matthew T. Cole*
(University College Dublin)

Working Paper: Comments Welcome

June 4, 2010

Abstract

Since firm heterogeneity has been introduced into international trade models, the importance of firm entry and exit (the extensive margin) has been highlighted. Thomas Chaney (2008) illustrates how accounting for heterogenous firms (and this extensive margin) alters the standard gravity equation. In particular, it reverses the previously predicted effect the elasticity of substitution has on the elasticity of trade flows. Further, Chaney shows that the elasticity of trade flows with respect to variable trade costs is a constant. As is common, iceberg transport costs are used as the variable trade barrier. However, in many empirical studies, *ad valorem* tariffs are also used as a form of trade barrier, which as Cole (2010) points out, is not isomorphic to iceberg transport cost in a monopolistically competitive setting. In this comment, I solve the Chaney (2008) model using *ad valorem* tariffs instead of iceberg transport costs and show the elasticity of trade flows with respect to tariffs is *not* constant, but depends on the elasticity of substitution.

JEL classification: F10; F12; F17

Keywords: Intra-industry Trade; Gravity; Firm heterogeneity; Monopolistic competition

*Corresponding author: School of Economics, University College Dublin, Newman Building (Room G215), Belfield, Dublin 4 Ireland. Email: matthew.cole@ucd.ie

1 Introduction

By introducing firm heterogeneity and fixed costs of exporting, Thomas Chaney (2008) is able to reverse the predictions of the classic Krugman (1980) model. Specifically, Krugman (1980) predicts that a higher elasticity of substitution between goods magnifies the impact of trade barriers on trade flows, where Chaney (2008) shows that the effect on trade flows is actually dampened by higher levels of elasticity of substitution.[1] The reasoning behind this reversal is driven by how the elasticity of substitution affects both the intensive and extensive margins. That is, the intensive margin is more sensitive but the extensive margin is less sensitive with a higher level of elasticity of substitution. Furthermore, with respect to variable trade costs, Chaney finds that the affect on the intensive and extensive margins exactly cancel out. It is this result in which I will provide clarification.

Chaney, as is fairly standard in the literature, uses iceberg transport costs to represent variable trade costs. However, as Cole (2010) illustrates, in a model of monopolistic competition, iceberg transport costs are not isomorphic to *ad valorem* tariffs, particularly with respect to the extensive margin. The difference lies in how the two barriers affect the level of firm profits. Iceberg transport costs are measured in lost output, but since firms charge a markup over marginal costs, a portion of this loss is recouped from the consumer. However, an *ad valorem* tariff is charged on the price (including the firm's markup). Thus, the entire cost is bore by the firm. Interestingly, the price charged is identical under both restrictions, but since the profit level is lower under a tariff, the firm cutoff is different. This difference is seen through the extensive margin and the price index.

My main point here is that it matters how one chooses to model trade barriers. It is not my intention to declare one better than the other; it is clear that countries charge tariffs *and* it is costly to transport goods. Thus, if you have an empirical specification including both trade barriers, it would be prudent to take care in how one models each type. In particular,

[1] Chaney (2008) makes the further assumption that the productivity across firms is distributed Pareto, which is close to the observed size distribution of US firms.

the researcher may want to interact the elasticity of substitution with tariffs, while this may be unnecessary with transport costs.[2] The rest of the paper proceeds as follows. Section 2 reminds the reader of the Chaney (2008) model setup and points out where using *ad valorem* tariffs alters this setup. Section 3 introduces trade and finds an altered gravity equation. Section 4 breaks down the effects of the intensive and extensive margins. Section 5 concludes.

2 Setup

I follow Chaney (2008) very closely, maintaining notation and setup, with two main exceptions. Since iceberg transport costs are different than *ad valorem* tariffs, in order to prevent confusion, I represent tariffs on goods shipped from country i to country j in sector h as $t_{ij}^h > 1$ instead of Chaney's use of τ_{ij}^h. Secondly, I approach where a consumer receives her income in a slightly altered way. It is inherent to the iceberg transport cost assumption that output (and income) is lost to the economy whereas tariffs create revenue for the government. I could simply have the government throw the revenue away, but as Cole (2010) highlights, firm profits are lower when faced with *ad valorem* tariffs then iceberg transport costs. Further, since Chaney (2008) uses a clever method of distributing a share global firm profits back to consumers, using a different trade barrier would affect income and thus consumer demand differently. Therefore, I impose that the government contribute a fraction of tariff revenue to this global fund (which ensures the dividend income takes the same form as in Chaney (2008)) and throw away the rest.

There are N potentially asymmetric countries that produce goods using only labor. Country n has a population of L_n. Consumers in each country maximize utility derived from the consumption of goods from $H+1$ sectors. Sector 0 provides a single homogenous good. The other H sectors are made of a continuum of differentiated goods. If a consumer consumes q_o units of good 0, and $q_h(\omega)$ units of each variety ω of good h, for all varieties in the set Ω_h

[2] In fact, modeling simple transport costs is not as straightforward. Irarrazabal, Moxnes, and Opromolla (2010) show in a model that allows for both iceberg and per-unit costs that the pure iceberg model is rejected.

(determined in equilibrium), she gets a utility U,

$$U \equiv q_o^{\mu_0} \prod_{h=1}^{H} \left(\int_{\Omega_h} q_h(\omega)^{(\sigma_h - 1)/\sigma_h} d\omega \right)^{[\sigma_h/(\sigma_h - 1)]}, \tag{1}$$

where $\mu_0 + \sum_{h=1}^{H} \mu_h = 1$, and where $\sigma_h > 1$ is the elasticity of substation between two varieties of good h.

2.1 Trade Barriers and Technology

As in Chaney (2008), there are two types of trade barriers, a variable (t_{ij}^h) and a fixed cost (f_{ij}^h). However, in my model, variable trade costs take the form of an *ad valorem* tariff and not iceberg transport cost. Each firm in sector h draws a random unit labor productivity φ. The unit labor productivity is φ. The cost of producing q units of a good and selling them in country j for a firm with productivity φ is

$$C_{ij}^h(q) = \frac{w_i}{\varphi} q + f_{ij}^h. \tag{2}$$

The price is the usual constant markup,[3]

$$p_{ij}^h(\varphi) = \frac{\sigma_h}{(\sigma_h - 1)} \frac{w_i}{\varphi},$$

I assume that productivity shocks are drawn from a Pareto distribution with shape parameter γ_h: productivity is distributed over $[1, +\infty)$ according to

$$P(\tilde{\varphi}_h < \varphi) = G_h(\varphi) = 1 - \varphi^{-\gamma_h}, \tag{3}$$

with $\gamma_h > \sigma_h - 1$.

[3]Note that the *ad valorem* tariff t_{ij}^h is not included as that is imposed on the consumer. This is done for clarity; it does not matter whether the tax is imposed on the firm or the consumer – see Cole (2010).

2.2 Demand for Differentiated Goods

The total income spent by workers in country j, Y_j, is the sum of their labor income ($w_j L_j$) and of the dividends they get from their portfolio ($w_j L_j \pi$), where π is the dividend per share of the global mutual fund. Note, that unlike Chaney (2008), where the global mutual fund is made up solely of firm profits, my fund is comprised of firm profits and a fraction of tariff revenue, (κ_{ij}^h). Exports from country i to country j in sector h, by a firm with a labor productivity φ, are[4]

$$x_{ij}^h(\varphi) = p_{ij}^h(\varphi) q_{ij}^h(\varphi) = \mu_h Y_j t_{ij}^{-\sigma} \left(\frac{p_{ij}^h(\varphi)}{P_j^h} \right)^{1-\sigma_h} \tag{4}$$

where P_j^h is the ideal price index for good h in country j. If only those firms above the productivity threshold $\bar{\varphi}_{kj}^h$ in country k and sector h export to country j, the ideal price index for good h in country j, P_j, and dividends and tariff refund per share, π, are defined as

$$P_j^h = \left(\sum_{k=1}^{N} w_k L_k \int_{\bar{\varphi}_{kj}^h}^{\infty} \left(\frac{\sigma_h}{(\sigma_h - 1)} \frac{w_k t_{kj}}{\varphi} \right)^{1-\sigma_h} dG_h(\varphi) \right)^{1/(1-\sigma_h)} \tag{5}$$

$$\pi = \frac{\sum_{h=1}^{H} \sum_{k,l=1}^{N} w_k L_k \left(\int_{\bar{\varphi}_{kl}^h}^{\infty} \pi_{kl}^h(\varphi) + \kappa_{kl}^h(\varphi) dG(\varphi) \right)}{\sum_{n=1}^{N} w_n L_n} \tag{6}$$

where

$$\begin{aligned} \pi_{kl}^h(\varphi) &= \left(\frac{\sigma_h}{(\sigma_h - 1)} \frac{w_i}{\varphi} - \frac{w_i}{\varphi} \right) q_{kl}^h(\varphi) - f_{kl}^h \\ &= \left(\frac{1}{(\sigma_h - 1)} \right) \frac{w_i}{\varphi} q_{kl}^h(\varphi) - f_{kl}^h \end{aligned}$$

are the net profits that firm with productivity φ in country k and sector h earns from exporting to country l, and

$$\kappa_{kl}^h = \frac{(t_{kl} - 1) p_{kl}^h(\varphi) q_{kl}^h(\varphi)}{\sigma_h}$$

[4]This will be different than Chaney because the tariff revenue stays in country j.

is the portion of tariff revenue contributed to the global mutual fund. Adding this extra portion makes the π in my model identical to the π in Chaney's paper.

Following Chaney, I only consider sector h and drop the h superscript for the next section.

3 Trade with Heterogeneous Firms

In this section, I characterize the equilibrium with trade.

3.1 Productivity Threshold

The profits firm φ earns when exporting from i to j are[5]

$$\pi_{ij} = \frac{\mu Y_j t_{ij}^{-\sigma}}{\sigma} \left[\frac{\sigma}{(\sigma-1)} \frac{w_i}{\varphi P_j} \right]^{1-\sigma} - f_{ij}.$$

Define the threshold $\bar{\varphi}_{ij}$ from $\pi_{ij}(\bar{\varphi}_{ij}) = 0$ as the productivity of the least productive firm in country i able to export to country j:

$$\bar{\varphi}_{ij} = \lambda_1 \left(\frac{f_{ij} t_{ij}^{\sigma}}{Y_j} \right)^{\frac{1}{(\sigma-1)}} \frac{w_i}{P_j} \tag{7}$$

where λ_1 is a constant.[6]

3.2 Equilibrium Price Indices

Note that $Y_k = w_k L_k (1+\pi)$ so $w_k L_k = \frac{Y_k}{(1+\pi)}$. Thus, the price index is

$$P_j = \lambda_2 Y_j^{\frac{(\sigma-1)-\gamma}{\gamma(\sigma-1)}} \theta_j \tag{8}$$

[5]Note that this is slightly different than the profit function with iceberg transport costs, which is

$$\pi_{ij} = \frac{\mu Y_j}{\sigma} \left[\frac{\sigma}{(\sigma-1)} \frac{w_i \tau_{ij}}{\varphi P_j} \right]^{1-\sigma} - f_{ij}.$$

[6]$\lambda_1 = \left(\frac{\sigma}{\sigma-1}\right) \left(\frac{\sigma}{\mu}\right)^{1/(\sigma-1)}$

where

$$\begin{aligned}\lambda_2^{\gamma} &= \left(\frac{\gamma-(\sigma-1)}{\gamma}\right)\left(\frac{\sigma}{\mu}\right)^{\frac{\gamma-(\sigma-1)}{(\sigma-1)}}\left(\frac{\sigma}{(\sigma-1)}\right)^{\gamma}\left(\frac{1+\pi}{Y}\right)\\ \theta_j^{-\gamma} &= \sum_{k=1}^{N}\left(\frac{Y_k}{Y}\right)w_k^{-\gamma}t_{kj}^{1+\frac{\sigma\gamma}{1-\sigma}}f_{kj}^{1+\frac{\gamma}{1-\sigma}}.\end{aligned}$$

3.3 Equilibrium Exports, Thresholds, and Profits

Plugging the general equilibrium price index from equation (8) into the demand function, and into the productivity threshold from equation (7), I can solve for firm level exports, the productivity thresholds and total world profits.

$$\begin{aligned}x_{ij}(\varphi) &= \begin{cases}\lambda_3\left(\frac{Y_j}{Y}\right)^{\frac{(\sigma-1)}{\gamma}}t_{ij}^{-\sigma}\left(\frac{\theta_j}{w_i}\right)^{\sigma-1}\varphi^{\sigma-1}, \text{ if } \varphi\geq\bar{\varphi}_{ij}\\ 0 \text{ otherwise,}\end{cases} \qquad (9)\\ \bar{\varphi}_{ij} &= \lambda_4\left(\frac{Y}{Y_j}\right)^{\frac{1}{\gamma}}\left(\frac{w_i}{\theta_j}\right)\left(f_{ij}t_{ij}^{\sigma}\right)^{\frac{1}{(\sigma-1)}}\\ Y_i &= (1+\lambda_5)w_iL_i\\ \pi &= \lambda_5\end{aligned}$$

where λ_3, λ_4, and λ_5 are constants and identical those of Chaney (2008).[7] Though these constants are identical to Chaney, note that the equilibrium firm level exports and productivity thresholds are slightly different. These differences translate into an altered Proposition 1:

[7]

$$\begin{aligned}\lambda_3 &= \sigma\lambda_4^{1-\sigma}\\ \lambda_4 &= \left[\left(\frac{\sigma}{\mu}\right)\left(\frac{\gamma}{\gamma-(\sigma-1)}\right)\frac{1}{(1+\lambda_5)}\right]^{\frac{1}{\gamma}}\\ \lambda_5 &= \frac{\sum_{h=1}^{H}\left(\frac{\sigma_h-1}{\gamma_h}\right)\frac{\mu_h}{\sigma_h}}{1-\sum_{h=1}^{H}\left(\frac{\sigma_h-1}{\gamma_h}\right)\frac{\mu_h}{\sigma_h}}\end{aligned}$$

Proposition 1. *Total (f.o.b.) X_{ij}^h in sector h from country i to country j are given by*

$$X_{ij}^h = \mu_h \left(\frac{Y_i Y_j}{Y}\right) \left(\frac{w_i t_{ij}^{h\,\frac{\sigma}{\sigma-1}}}{\theta_j}\right)^{-\gamma_h} f_{ij}^{-\left[\frac{\gamma_h}{(\sigma_h-1)}-1\right]}. \tag{10}$$

Exports are a function of country sized (Y_i and Y_j), workers' productivity (w_i), the bilateral trade costs variable (t_{ij}^h) and fixed (f_{ij}^h), and the measure of j's remoteness from the rest of the world (θ_j^h).

Proof. See the Appendix □

We can see here that many of the points Chaney makes still ring true, particularly with how the level of heterogeneity parameter (γ) affects the gravity equation. However, his third point does not hold when trade barriers are *ad valorem* tariffs. That is, the elasticity of total exports with respect to variable cost *does* depend on the elasticity of substitution between goods, σ. This difference will be dealt with in more detail in the next section.

4 Intensive versus Extensive Margins of Trade

The next difference between using *ad valorem* tariffs instead of iceberg transport costs is illustrated by Proposition 2 and is driven by the effect on the extensive margin. Chaney shows that the elasticity of trade flows with respect to variable trade costs is 0. I illustrate here that this is not true for all variable trade costs; i.e. tariffs.

Proposition 2. *The elasticity of substitution (σ) has a negative effect on the elasticity of trade flows with respect to ad valorem tariffs (ζ) and fixed costs (ξ):*

$$\text{if } \zeta \equiv -\frac{d\ln X_{ij}}{d\ln t_{ij}} \text{ and } \xi \equiv \frac{d\ln X_{ij}}{d\ln f_{ij}}, \text{ then } \frac{\partial\zeta}{\partial\sigma} < 0 \text{ and } \frac{\partial\xi}{\partial\sigma} < 0$$

Proof. The proof for the result pertaining to fixed costs (ξ) is identical to that of Chaney (2008); thus I omit it and impose $df_{ij} = 0$. Differentiating the expression for aggregate

exports, $X_{ij} = w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} x_{ij}(\varphi) dG(\varphi)$, I get the following expression for each margin:

$$dX_{ij} = \underbrace{\left(w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial t_{ij}} dG(\varphi) \right)}_{\text{Intensive margin}} dt_{ij} - \underbrace{\left(w_i L_i x(\bar{\varphi}_{ij}) G'(\bar{\varphi}_{ij}) \frac{\partial \bar{\varphi}_{ij}}{\partial t_{ij}} \right)}_{\text{Extensive margin}} dt_{ij}$$

In elasticity notation, I get the following expression for each margin for changes in tariffs, t_{ij}:[8]

$$\zeta \equiv -\frac{d \ln X_{ij}}{d \ln t_{ij}} = \underbrace{\sigma}_{\text{Intensive}} + \underbrace{\frac{\sigma\gamma}{\sigma - 1} - \sigma}_{\text{Extensive}} = \frac{\sigma\gamma}{\sigma - 1}.$$

The effect of σ on each margin no longer cancels out and we are left with:

$$\frac{\partial \zeta}{\partial \sigma} = \frac{-\gamma}{(\sigma - 1)^2} < 0$$

□

The intuition here is the same as Chaney (2008), the extensive margin becomes less sensitive as the elasticity of substitution increase counteracting the effects of σ on the intensive margin. However, if trade barriers are *ad valorem* tariffs, these effects do not exactly cancel out; the effect on the extensive margin outweighs that of the intensive margin.

5 Conclusion

Chaney (2008), quite elegantly, illustrates how firm heterogeneity can alter the standard gravity equation. This alteration is driven by the extensive margin effect; i.e. the entry and exit decision of firms. Cole (2010) illustrates that this extensive margin is sensitive to the

[8] See the Appendix for a complete derivation.

type of trade barrier. I show in this comment that the elasticity of trade flows with respect to *ad valorem* tariffs is a function of the elasticity of substitution and not a constant. This is interesting in its own right theoretically, but has actual implications empirically. That is, it the coefficients should not be expected to be the same and more importantly, the coefficient on tariffs should be dependent on the elasticity of substitution.

APPENDIX

Proposition A1. *(Reminded) Total (f.o.b.)* X_{ij}^h *in sector* h *from country* i *to country* j *are given by*

$$X_{ij}^h = \mu_h \left(\frac{Y_i Y_j}{Y}\right) \left(\frac{w_i t_{ij}^{h\,\frac{\sigma}{\sigma-1}}}{\theta_j}\right)^{-\gamma_h} f_{ij}^{-\left[\frac{\gamma_h}{(\sigma_h-1)}-1\right]}.$$

Exports are a function of country sized (Y_i *and* Y_j*), workers' productivity (*w_i*), the bilateral trade costs variable (*t_{ij}^h*) and fixed (*f_{ij}^h*), and the measure of* j*'s remoteness from the rest of the world (*θ_j^h*).*

Proof. Aggregate Exports in sector h from country i to country j is

$$X_{ij}^h = w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} x_{ij}^h(\varphi) dG(\varphi).$$

Using the specific assumption about the distribution G, this becomes

$$X_{ij}^h = w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} \lambda_3^h \left(\frac{Y_j}{Y}\right)^{\frac{(\sigma_h-1)}{\gamma_h}} (t_{ij}^h)^{-\sigma_h} \left(\frac{\theta_j^h}{w_i}\right)^{\sigma_h-1} \varphi^{\sigma_h-1} \frac{\varphi^{-\gamma_h-1}}{\gamma_h} d\varphi$$

with

$$\bar{\varphi}_{ij}^h = \lambda_4^h \left(\frac{Y}{Y_j}\right)^{\frac{1}{\gamma_h}} \left(\frac{w_i}{\theta_j^h}\right) \left(f_{ij} t_{ij_h}^{h\,\sigma}\right)^{\frac{1}{(\sigma_h-1)}}$$

Solving this integral yields:

$$
\begin{aligned}
X_{ij}^h &= w_i L_i \lambda_3^h \left(\frac{Y_j}{Y}\right)^{\frac{(\sigma_h-1)}{\gamma_h}} \left(\frac{\theta_j}{w_i}\right)^{\sigma_h-1} \frac{t_{ij}^{-\sigma_h}\gamma_h}{\gamma_h-(\sigma_h-1)} \left[\lambda_4 \left(\frac{Y}{Y_j}\right)^{\frac{1}{\gamma_h}} \left(\frac{w_i}{\theta_j}\right) \left(f_{ij} t_{ij}^{\sigma_h}\right)^{\frac{1}{(\sigma_h-1)}}\right]^{(\sigma_h-1)-\gamma_h} \\
&= w_i L_i \lambda_3^h \left(\frac{Y_j}{Y}\right) t_{ij}^{\frac{-\sigma_h\gamma_h}{\sigma_h-1}} \left(\frac{\theta_j}{w_i}\right)^{\gamma_h} \frac{\gamma_h}{\gamma_h-(\sigma_h-1)} \left[\lambda_4^h \left(f_{ij}\right)^{\frac{1}{(\sigma_h-1)}}\right]^{(\sigma_h-1)-\gamma_h} \\
&= \lambda_3^h (\lambda_4^h)^{(\sigma_h-1)-\gamma_h} \frac{\gamma_h}{\gamma_h-(\sigma_h-1)} \left(\frac{w_i L_i Y_j}{Y}\right) t_{ij}^{\frac{-\sigma_h\gamma_h}{\sigma_h-1}} \left(\frac{w_i}{\theta_j^h}\right)^{-\gamma_h} f_{ij}^{\frac{(\sigma_h-1)-\gamma_h}{(\sigma_h-1)}} \\
&= \sigma_h (\lambda_4)^{-\gamma_h} \frac{\gamma_h}{\gamma_h-(\sigma_h-1)} \left(\frac{w_i L_i Y_j}{Y}\right) (t_{ij}^h)^{\frac{-\sigma_h\gamma_h}{\sigma_h-1}} \left(\frac{w_i}{\theta_j^h}\right)^{-\gamma_h} f_{ij}^{\frac{(\sigma_h-1)-\gamma_h}{(\sigma_h-1)}} \\
&= \mu_h (1+\lambda_5^h) \left(\frac{w_i L_i Y_j}{Y}\right) t_{ij}^{\frac{-\sigma_h\gamma_h}{\sigma_h-1}} \left(\frac{w_i}{\theta_j}\right)^{-\gamma_h} f_{ij}^{\frac{(\sigma_h-1)-\gamma_h}{(\sigma_h-1)}} \\
&= \mu_h \left(\frac{Y_i Y_j}{Y}\right) t_{ij}^{\frac{-\sigma_h\gamma_h}{\sigma_h-1}} \left(\frac{w_i}{\theta_j}\right)^{-\gamma_h} f_{ij}^{\frac{(\sigma_h-1)-\gamma_h}{(\sigma_h-1)}}
\end{aligned}
$$

□

Proposition A2. *(Reminded) The elasticity of substitution (σ) has a negative effect on the elasticity of trade flows with respect to ad valorem tariffs (ζ) and fixed costs (ξ):*

$$\text{if } \zeta \equiv -\frac{d\ln X_{ij}}{d\ln t_{ij}} \text{ and } \xi \equiv \frac{d\ln X_{ij}}{d\ln f_{ij}}, \text{ then } \frac{\partial\zeta}{\partial\sigma} < 0 \text{ and } \frac{\partial\xi}{\partial\sigma} < 0$$

Proof.

$$dX_{ij} = \underbrace{\left(w_i L_i \int_{\bar\varphi_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial t_{ij}} dG(\varphi)\right)}_{\text{Intensive margin}} dt_{ij} - \underbrace{\left(w_i L_i x(\bar\varphi_{ij}) G'(\bar\varphi_{ij}) \frac{\partial\bar\varphi_{ij}}{\partial t_{ij}}\right)}_{\text{Extensive margin}} dt_{ij}$$

Using the definition of equilibrium individual exports from equation (9), and assuming that country i is small enough and/or remote enough, so that $\partial\theta_j/\partial t_{ij} \approx 0$, we get

$$\frac{\partial x_{ij}(\varphi)}{\partial t_{ij}} = -\sigma \frac{x_{ij}(\varphi)}{\partial t_{ij}}.$$

Integrating over all exporters, we get

$$
\begin{aligned}
\textit{Elasticity of the intensive margin w.r.t. tariffs} &= -\frac{t_{ij}}{X_{ij}} \left(w_i L_i \int_{\bar\varphi_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial t_{ij}} dG(\varphi)\right) \\
&= \sigma \frac{t_{ij}}{X_{ij}} \frac{w_i L_i \int_{\bar\varphi_{ij}}^{\infty} x_{ij}(\varphi) dG(\varphi)}{t_{ij}} \\
&= \sigma \frac{t_{ij}}{X_{ij}} \frac{X_{ij}}{t_{ij}} \\
&= \sigma.
\end{aligned}
$$

Now, define $x_{ij} = \lambda_{ij}\varphi^{\sigma-1}$ and note that $G'(\varphi) = \varphi^{-\gamma-1}/\gamma$. Aggregate exports can be written in the following way

$$\begin{aligned} X_{ij} &= w_i L_i \lambda_{ij} \int_{\bar{\varphi}_{ij}}^{\infty} \varphi^{\sigma-1}\gamma\varphi^{-\gamma-1} \\ &= \frac{\gamma}{\gamma-(\sigma-1)} w_i L_i \lambda_{ij} \bar{\varphi}^{(\sigma-1)-\gamma} \\ &= \frac{1}{\gamma-(\sigma-1)} w_i L_i x_{ij}(\bar{\varphi}) G'(\bar{\varphi})\bar{\varphi} \end{aligned}$$

We therefore get the simple solution for the elasticity:

$$\begin{aligned} \textit{Elasticity of the extensive margin w.r.t. tariffs} &= \frac{t_{ij}}{X_{ij}}\left(w_i L_i x_{ij}(\bar{\varphi}) G'(\bar{\varphi})\frac{\partial\bar{\varphi}}{\partial t_{ij}}\right) \\ &= \frac{t_{ij}}{X_{ij}}\left(\frac{w_i L_i x_{ij}(\bar{\varphi}) G'(\bar{\varphi})\bar{\varphi}}{t_{ij}}\frac{\sigma}{\sigma-1}\right) \\ &= (\gamma-(\sigma-1))\frac{t_{ij}}{X_{ij}}\left(\frac{X_{ij}}{t_{ij}}\frac{\sigma}{\sigma-1}\right) \\ &= \frac{\sigma\gamma}{\sigma-1} - \sigma \end{aligned}$$

□

References

[1] Chaney, T. (2008). "Distorted Gravity: The Intensive and Extensive Margins of International Trade." *American Economic Review*, 98(4), 1707-1721.

[2] Cole, M.T. (2010). "Not All Trade Restricitons are Created Equally." University College Dublin Working Paper No. 201009.

[3] Irarrazabal, A., A. Moxnes, and L. Opromolla (2010). "The Tip of the Iceberg: Modeling Trade Costs and implications for Intra-Industry Reallocation." CEPR Discussion Paper No. DP7685.